

## Chapter 10 – Combined Operations on C, D, CI, DI, CD, DF, CIF, A, B, BI, K and K' Scales

In this unit we will show how the order of operations and the selection of scales can greatly reduce the moves required. The effect of this is to increase accuracy, speed and the range of problems you can handle on your Slide Rule.

Note: The A and B scales or the K and K' scales (if you Slide Rule has a K' Scale), can be used for multiplication and division. On the A and B scales the 1, 10, 100 can be read as an index, while the K and K' scales 1, 10, 100, or 1,000 can be used as an index in calculations. The BI scale can be used in conjunction the A and B scales in the same way as the CI with the C and D scales, or the CIF with the CF and DF scales.

### 10.1 Simple Combinations or Roots, Powers and Reciprocals.

Example 1:  $\frac{1}{29.6^2} = 0.00114$

1. Set the hair line over 29.6 on the CI (or DI) scale. (Note under the hair line on the C (or D) scale we have the reciprocal of 29.6, but we need not read this).
2. Under the hair line read off '144' on the B (or A) scale as the answer.

$$\begin{aligned} \text{(i.e. approx)} &= \frac{1}{30^2} = \left(\frac{1}{30}\right)^2 \\ &= (.03)^2 \\ &= 0.0009 \end{aligned}$$

Therefore the answer is 0.00114

Note: We find the reciprocal of 29.6 first and then the square. This is possible because  $\frac{1}{29.6^2} = \left(\frac{1}{29.6}\right)^2$ .

If we were to use the reverse order of operations, it would necessitate reading the square of 29.6 off the A or B scale and transferring this value onto the C or CI scale to obtain the reciprocal, which of course is unsatisfactory.

Example 2:  $\frac{1}{29.6^3} = 0.0000386$

1. Set the hair line over 29.6 on the CI (or DI) scale.
2. Under the hair line read off 0.0000386 on the K (or K') scale as the answer.

Example 3:  $\frac{1}{\sqrt{29.6}} = 0.1838$

1. Set the hair line over 29.6 on the B (or A) scale. (Note: under the hair line on the C (or D) scale we have the square root of 29.6, but we need not read this).
2. Under the hair line read off 0.1838 on the CI (or DI) scale as the answer.

Example 4:  $\frac{1}{3\sqrt{29.6}} = 0.3235$

1. Set the hair line over 29.6 on the K (or K') scale.
2. Under the hair line read off 0.3235 on the CI (or DI) scale as the answer.

Example 5:  $3\sqrt{29.6^2} = 9.57$

1. Set the hair line over 29.6 on the K (or K') scale. (Note: under the hair line on the C (or D) scale we have the cube root of 29.6, but we need not read this.)
2. Under the hair line read off 9.57 on the A (or B) scale as the answer.

Example 6:  $\sqrt{29.6^3} = 161$

1. Set the hair line over 29.6 on the A (or B) scale.
2. Under the hair line read off 161 on the K (or K') scale as the answer.

Example 7:  $29.6^4 = 768,000$

1. Set the hair line over 29.6 on the D scale.
2. Place the 29.6 on the CI scale below the hair line.
3. Above the right index of the B scale read off 768,000 on the A scale as the answer. (Note: the left index of the B scale will sometimes be used instead of the right index.)

OR

1. Set the hair line over 29.6 on the K scale.
2. Place the 29.6 of the CI scale below the hair line.
3. Above the right index of the CI scale read off 768,000 on the K scale as the answer.

Example 8:  $29.6^5 = 22,750,000$

1. Set the hair line over 2.6 on the D scale.
2. Place the 29.6 of the CI scale below the hair line.
3. Reset the hair line over 29.6 on the B scale.
4. Under the hair line read off 22,750,000 on the A scale as the answer.

Example 9:  $29.6^6 = 673,000,000$

1. Place the left index of the C scale over the 29.6 on the D scale.
2. Set the hair line over 29.6 on the C scale.
3. Under the hair line read off 673,000,000 on the K scale as the answer.

Example 10:  $\frac{1}{29.6^4} = 0.0000013$

1. Place the 29.6 of the C scale over the left index of the D scale.
2. Set the hair line over 29.6 on the CI scale.
3. Under the hair line read off 0.0000013 on the A scale as the answer.

Note: The last two steps in the above example could be done as follows:

2. Set the hair line over 29.6 on the D scale.
3. Under the hair line read off 0.0000013 on the BI scale as the answer.

**Exercise 10(a)**

- |                         |                                 |                          |
|-------------------------|---------------------------------|--------------------------|
| (i) $\frac{1}{6.8^2}$   | (v) $\frac{1}{\sqrt{70}}$       | (x) $\sqrt{61^3}$        |
| (ii) $\frac{1}{0.47^2}$ | (vi) $\frac{1}{\sqrt{5.4}}$     | (xi) $\sqrt[3]{96.1^2}$  |
| (iii) $\frac{1}{34^2}$  | (vii) $\frac{1}{\sqrt[3]{143}}$ | (xii) $\sqrt[3]{0.85^2}$ |
| (iv) $\frac{1}{1.4^2}$  | (viii) $\frac{1}{3\sqrt{0.54}}$ | (xiii) $2.3^4$           |
|                         | (ix) $\sqrt{13.5^3}$            | (xiv) $49.5^4$           |
|                         |                                 | (xv) $6.7^5$             |
|                         |                                 | (xvi) $19.5^5$           |
|                         |                                 | (xvii) $0.9^6$           |
|                         |                                 | (xviii) $4.1^6$          |

(xix)  $\frac{1}{8.3^4}$

(xx)  $\frac{1}{0.674^4}$

### 10.2 Continued Multiplication and Division

In this section we will combine the basic methods of multiplication and division of Units 2, 3, 4 and 8. You may find it helpful to review these briefly before proceeding with this section.

Example 1:  $12.4 \times 8.4 \times 0.157 = 16.35$

1. Set the hair line over 12.4 on the D scale.
2. Place the 8.4 of the CI scale under the hair line.

Note: the progressive answer is marked on the D scale by the left index of the C scale.

3. Reset the hair line over the 0.157 on the C scale.
4. Under the hair line read off 16.35 on the D scale as the answer.

Example 2:  $2.32 \times 60.5 \div 0.082 = 1710$

1. Set the hair line over 2.32 on the D scale.
2. Place the 60.6 on the CI scale below the hair line.

Note: the progressive answer is marked on the D scale by the left index of the C scale.

3. Reset the hair line over the 0.082 on the CI scale.
4. Under the hair line read off 1710 on the D scale as the answer.

Example 3:  $7.5 \div 4.8 \times 30.4 = 47.5$

1. Set the hair line over 7.5 on the D scale.
2. Place the 4.8 on the C scale under the hair line.

Note the progressive answer is marked on the D scale by the left index of the C scale.

3. Reset the hair line over 30.4 on the C scale.
4. Under the hair line read off 47.5 on the D scale as the answer.

Example 4:  $36.6 \div 0.71 \div 2.26 = 22.8$

1. Set the hair line over 36.6 on the D scale.
2. Place the 0.71 on the C scale under the hair line.

Note the progressive answer is marked on the D scale by the right index of the C scale.

3. Reset the hair line over 2.26 on the CI scale.
4. Under the hair line read off 22.8 on the D scale as the answer.

Note: In combination multiplication and division rules are:

- (a) When multiplication comes first, always do it by using the D and CI scales.
- (b) When division comes first, always do it by using the C and D scales.

The reason for this is that when we multiply by the CI and D scales, or divide by the C and D scales, the answer is always marked on the D scale by the left or right index of the C scale, thus allowing a further multiplication or division without moving the slide. Hence, if the second operation is multiplication, we use the C and D scales, and if the second operation is a division we use the CI and D scales. (Check these points carefully in the four examples above.)

Example 5:  $3.35 \times 47 \div 25.9 \div 41 \times 8.85 = 1.312$

1. Set the hair line over 3.35 on the D scale.
2. Place the 47 of the CI scale under the hair line.
3. Reset the hair line over 25.9 on the CI scale.
4. Place the 41 of the C scale under the hair line.
5. Reset the hair line over 8.85 of the CF scale.
6. Under the hair line read off 1.38 on the DF scale as the answer.

(Note: in the above example we switched to the Folded scales for the last multiplication, as 8.85 on the C scale was off the end of the D scale. Otherwise it would have required the slide to have been moved.)

Exercise 10(b)

- |                                     |  |
|-------------------------------------|--|
| (i) $43 \times 5.1 \times 0.12 =$   | (ix) $6.4 \times 3.5 \times 18 \times 0.14 =$  |
| (ii) $52 \times 720 \times 0.041 =$ | (x) $2.46 \times 52.3 \div 25.2 \div 3.6 =$    |
| (iii) $2.4 \times 32 \div 1.94 =$   | (xi) $2.46 \times 52.3 \div 25.2 \times 3.6 =$ |
| (iv) $156 \times 6.57 \div 92.1 =$  | (xii) $6.3 \div 1.3 \times 2.3 \times 4.9 =$   |
| (v) $19 \div 6.4 \div 15 =$         | (xiii) $70.6 \div 4.32 \div 15.25 \div 1.16 =$ |
| (vi) $16.4 \div 3.14 \div 2.75 =$   | (xiv) $39.4 \div 1.73 \div 12.4 \times 8.66 =$ |
| (vii) $11 \div 73 \div 830 =$       | (xv) $8.3 \times 3.9 \times 20 \div 16 =$      |
| (viii) $38 \div 5.2 \times 4.75 =$  | (xvi) $28 \div 1.3 \times 5.6 \div 1.5 =$      |

**10.3 Multiplication and Division of Roots, Powers and Reciprocals**

There are too many possible combinations for us to cover ever type, but the following table gives many possibilities.

Note that often care must be taken in setting numbers on the A, B and K scales, as we will recall for example  $\sqrt{2}$  and  $\sqrt{20}$  are located at different positions on the A and B scale. Thus, numbers must be located on the A, B and K scales according to the rules given in Units 5 and 6. In some of the following, with certain numbers the answer may run off the end of the scale. In such cases it will be necessary to reset the slide unless the C and D scales are involved, when it will be possible to us the CF and DF scales and avoid a further movement of the slide. (In the following, 'H.L.' stands for Hair Line.)

Example	Set HL Over	Under HL Place	Reset HL over	Under HL answer
$ab^2$	a on A scale	Index of B scale	b on C scale	on A scale
$a^2b$	a D	Index C	b B	A
$ab^3$	a K	Index B	b C	K
$ab^4$	a A b D	b CI a BI	b C b C	A A
$a^2b^2$	a D a D	Index C b CI	b C Index C	A A
$a^2b^3$	a D	b CI	b B	A
$a^2b^2c$	c A a D	a CI c BI	b C b C	A A
$a^3b^3c$	c K	a CI	b C	K
$\frac{a}{b^2}$	a A a A	b C Index B	Index C b CI	A A
$\frac{a}{b^3}$	a K	b C	Index C	K
$\frac{a^2}{b^3}$	a D a D	b C b B	b BI b CI	A A
$\frac{a^2}{b}$	a D	b B	Index B	A
$\frac{a^3}{b}$	a D a D	b B b K'	a B index K'	A K
$\frac{a^3}{b^2}$	a A	a CI	b CI	A
$\frac{ab^2}{c^2}$	b D	c C	a B	A

$\frac{a}{b^2c^2}$	a A	b C	c CI	A
$\frac{a}{bc^2}$	a A	b B	c CI	A
$\frac{1}{ab}$	Index D	a C	b CI	D
$\frac{1}{ab^2}$	Index A	a B	b CI	A
$\frac{1}{a^2b^2}$	Index D	a C	b CI	A
$\frac{1}{a^2b^3}$	a K	b C	a CI	K
$\sqrt{ab}$	a A	Index B	b B	D
$\sqrt{abc}$	a A	b BI	c B	D
$a\sqrt{b}$	a D	Index C	b B	D
$a^2\sqrt{b}$	b A	a CI	a C	D
$\sqrt{\frac{a}{b}}$	a A	b B	Index B	D
$\frac{\sqrt{a}}{b}$	a A	b C	Index C	D
$\frac{a}{\sqrt{b}}$	a D	b B	Index C	D
$\sqrt{\frac{ab}{c}}$	a A	c B	b B	D
$\sqrt{\frac{a}{bc}}$	a A	b B	c BI	D
$\frac{a\sqrt{b}}{c}$	a A	b B	c BI	D
$\frac{\sqrt{a}}{bc}$	a A	b C	c C	D
$\frac{ab}{\sqrt{c}}$	a D	c B	b C	D
$ab\sqrt{c}$	c A	b CI	a C	D
$a\sqrt{bc}$	b A	a CI	c B	D
$a^3\sqrt{b}$	b K	a CI	Index CI	D
$\frac{\sqrt[3]{a}}{b}$	a K	b C	Index C	D

$\frac{a}{\sqrt[3]{b}}$	b K	a C	Index D	D
$\frac{\sqrt{a^3\sqrt{b}}}{c}$	b K	c C	a B	D
$\frac{a^3\sqrt{b}}{\sqrt{c}}$	b K	c B	a C	D
$\frac{1}{\sqrt{ab}}$	Index A a A	a B Index B	b A b B	CI DI
$\frac{1}{a\sqrt{b}}$	b A Index D	a CI a C	Index D b BI	C D
$a\pi\sqrt{b}$	b A	a CI	Index C	DF
$\frac{a\pi}{\sqrt{b}}$	a D	b B	Index C	DF
$\sqrt{a\pi}$	$\pi$ A	Index B	a B	D
$\frac{a\sqrt{b}}{\pi}$	b A	a CI	Index CF	D
$\frac{a}{\pi\sqrt{b}}$	a D	b B	Index CF	D
$a\sqrt{\frac{b}{\pi}}$	b A	$\pi$ B	a CF	DF

**Exercise 10(c)**

(i)  $47.9\sqrt{1.46}$

(ii)  $2.17 \times 8.63\sqrt{0.562}$

(iii)  $5.75^3\sqrt{69.8}$

(iv)  $\frac{\sqrt{6.45}}{8.27}$

(v)  $\frac{67.3}{\sqrt{123.4}}$

(vi)  $\frac{8.47\sqrt{63.7}}{14.64}$

(vii)  $\frac{15.7 \times 6.37}{\sqrt{127}}$

(viii)  $\frac{\sqrt{436}}{2.08 \times 5.63}$

(ix)  $\frac{1}{42.7\sqrt{2.63}}$

(x)  $2.46\sqrt{6.23 \times 4.97}$

(xi)  $\frac{17.22}{\sqrt{31.9 \times 20.6}}$

(xii)  $\frac{\sqrt{673}}{4.22\sqrt{51.5}}$

(xiii)  $40.2\sqrt{12.25^3\sqrt{106}}$

(xiv)  $\sqrt{32.6^3\sqrt{24.6}}$

(xv)  $27 \times 3.4^2$

(xvi)  $\frac{26.4^2}{31.2}$

(xvii)  $\frac{256}{12.7^2}$

(xviii)  $\frac{31.9 \times 16.3^2}{2.57}$

(xix)  $\frac{1}{\sqrt{6.3 \times 13.1}}$

(xxv)

(xx)  $\frac{1}{4.2\sqrt{6.1}}$

(xxi)  $\frac{1}{6.7^2 \times 3.1^3}$

(xxii)  $\sqrt{61\pi}$

(xxiii)  $\frac{61.4}{\pi\sqrt{19.7}}$

(xxiv)  $17.2\sqrt{\frac{29}{\pi}}$

#### 10.4 Miscellaneous Problems

##### Exercise 10(d)

(i)  $\left(\frac{24 \times 31.6}{68}\right)^2 =$

(ii)  $11.2\left(1 + \frac{39.4}{85}\right)^2 =$

(iii)  $\frac{2.9 \times 3.4^2}{6.1^2 - 4.4^2} =$

(iv)  $\frac{\sqrt{21} - \sqrt{15.4}}{\pi} =$

(v)  $2\left(\frac{29.8}{111} - \frac{3.21}{94.7}\right) =$

(vi)  $14.2^3\sqrt{6.4 + 1.96^2} =$

(vii)  $\frac{\sqrt{6.9^2 - 1}}{\sqrt{2.7^2 + 1}} =$

(viii)  $\frac{2.3^4 \pi}{132} =$

(ix)  $\frac{2500 \times 3.4 \times 32}{3 \times 10^6 \times \pi (1.7^4 - 1.4^4)} =$

(x)  $15 \times 0.43^2 \times 1.19^4 =$

(xi)  $\sqrt{13(13 - 6.4)(13 - 9.9)(13 - 9.7)}$

(xii)  $2\pi\sqrt{\frac{87.6}{980}} =$

(xiii)  $32^3 - \frac{(61 - 24)^2}{7.3} =$